

## MODELING OF FOREARM MYOELECTRIC SIGNALS

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### RESUMEN

La electromiografía (EMG) registra la actividad eléctrica de los músculos esqueléticos y desempeña un papel fundamental en rehabilitación, control de prótesis y diagnóstico de enfermedades neuromusculares. Sin embargo, su naturaleza no estacionaria y la presencia de ruido dificultan un modelado preciso. Existen métodos basados en modelos estocásticos, técnicas de aprendizaje automático y algoritmos de optimización, pero suelen requerir grandes volúmenes de datos y un elevado costo computacional, lo cual limita su aplicación en tiempo real. La identificación paramétrica, especialmente mediante el algoritmo de mínimos cuadrados recursivos, permite actualizar parámetros en tiempo real, adaptándose a la variabilidad y al ruido característicos de las señales EMG. No obstante, la implementación y validación en entornos reales enfrenta desafíos como la variabilidad inter- e intra-sujeto y la selección de la estructura del modelo. La motivación del presente trabajo es desarrollar un modelo matemático estructuralmente compuesto por 200 términos de funciones cosenoidales, exponenciales y de tipo raíz cuadrada, con el objetivo de reproducir una señal mioeléctrica adquirida experimentalmente con un error mínimo, mejorando así la precisión y la estabilidad del modelado bajo condiciones realistas.

Palabras clave: Electromiografía, Identificación paramétrica, Mínimos cuadrados recursivos

### ABSTRACT

Electromyography (EMG) records the electrical activity of skeletal muscles and plays a fundamental role in rehabilitation, prosthetic control, and neuromuscular disease diagnosis. Nevertheless, its nonstationary behavior and the presence of noise hinder accurate modeling. Existing approaches rely on stochastic models, machine learning techniques, and optimization algorithms; however, they often require large datasets and significant computational resources, which restrict their feasibility in real-time applications. Parametric identification, particularly through the recursive least squares algorithm, enables real-time parameter estimation by adapting to the variability and noise inherent in EMG signals. Nonetheless, real-world implementation and validation face challenges such as inter- and intra-subject variability and the selection of an appropriate model structure. The motivation of this work is to develop a mathematical model structurally composed of 200 terms involving

cosenoidal, exponential, and square root-type functions, with the objective of reproducing an experimentally acquired myoelectric signal with minimal error, thereby improving accuracy and stability of EMG modeling under realistic conditions.

Keywords: Electromyography, Parametric identification, Recursive least squares

### 1. INTRODUCCIÓN

Electromyography (EMG) has become a cornerstone for analyzing neuromuscular activity, with critical applications in biomedicine, rehabilitation engineering, and advanced human-machine interfaces. These signals, generated by the electrical activity of muscle fibers during contraction, encode valuable information about neuromuscular processes. Accurate modeling of EMG is therefore essential for improving the interpretation of muscle dynamics and for advancing the design of next-generation assistive and prosthetic devices [1].

The central objective of EMG signal modeling is not only to enhance the detection and classification of muscle activation patterns but also to optimize signal processing for real-time implementation [2]. Nevertheless, EMG signals are inherently nonstationary, nonlinear, and subject to noise, making them difficult to model with high fidelity. Additional challenges arise from inter- and intra-subject variability, muscle fatigue, and electromagnetic interference—factors particularly critical in forearm applications where multiple muscles are activated simultaneously.

To address these challenges, a wide spectrum of mathematical and computational frameworks has been investigated. Physics-based approaches, such as the Finite Element Method (FEM), reconstruct the spatial distribution of bioelectrical currents by solving Maxwell's equations, which under quasi-static assumptions reduce to Poisson formulations [3,4,5]. Signal-driven methods include Autoregressive (AR) models, where EMG signals are expressed as linear combinations of past values, and hybrid approaches such as the AR-Adaptive Kalman

Filter (AR-AKF), which integrate state-space formulations with adaptive filtering to suppress stochastic noise [6,7]. Adaptive identification techniques have also gained prominence, particularly the Recursive Least Squares (RLS) algorithm, which sequentially updates parameters with each new observation. Owing to its computational efficiency and adaptability, RLS is highly suited for real-time EMG analysis, offering greater precision and robustness compared to traditional models [8].

Recent developments further extend EMG modeling into stochastic and state-space domains. Lu et al. employed a Hidden Markov Model (HMM) combined with the Expectation-Maximization (EM) algorithm to capture temporal and probabilistic dynamics for hand manipulation recognition [9]. Furui et al. proposed a scale mixture stochastic model, in which EMG variance is treated as a latent variable following an inverse gamma distribution, thereby unifying Gaussian and non-Gaussian behaviors through a Student's t-distribution framework [10]. Dutra et al. developed a multivariable state-space model for continuous grasping-force estimation, combining RLS identification with Kalman filtering to enhance state estimation and noise rejection. Collectively, these approaches demonstrate an evolution from static and deterministic methods toward adaptive, probabilistic, and state-space frameworks better suited to the complexity of EMG signals [11].

Within this context, the present study contributes a parametric mathematical model composed of 200 structural terms—including cosenoidal, exponential, and square-root functions—whose coefficients are identified using the RLS algorithm. The model is designed to reproduce experimentally acquired EMG signals with minimal error, thereby improving stability and precision under realistic conditions. This work aims to strengthen the link between parametric identification and practical EMG applications, expanding its relevance to prosthetic control, rehabilitation, and human-machine interfacing.

This article is organized into three sections. Section 1 provides an overview of EMG signals and their relevance. Section 2 introduces the least squares methodology employed for parametric identification of the proposed model. Section 3 presents the experimental results and their validation.

## 2. PARAMETER IDENTIFICATION

This section outlines the parametric identification methodology employed for constructing dynamic models from experimentally acquired EMG signals. The theoretical

underpinnings of the Recursive Least Squares (RLS) algorithm are presented in detail, including formal mathematical derivations, while the necessary conditions for guaranteeing consistent and accurate parameter estimation of the underlying system are rigorously addressed.

### 2.1. Recursive Least Squares Algorithm

The Recursive Least Squares (RLS) algorithm is a well-established method for online parameter estimation in dynamic systems, valued for its computational efficiency and recursive update mechanism that incorporates new measurements in real time [8]. Its formulation originates from a linear regression framework, which can be expressed in compact matrix form as:

$$\mathbf{y}(k) = \boldsymbol{\varphi}^T(k)\boldsymbol{\theta} \quad (1)$$

Here,  $\mathbf{y}(k) \in \mathbb{R}^n$  denotes the observation vector corresponding to the discrete-time system outputs,  $\boldsymbol{\varphi}(k) \in \mathbb{R}^{p \times n}$  represents the regressor matrix constructed from known system functions, and  $\boldsymbol{\theta} \in \mathbb{R}^p$  is the parameter vector to be estimated.

This formulation establishes the foundation for a recursive estimation algorithm, enabling the iterative refinement of the parameter vector  $\boldsymbol{\theta}$  at each discrete time step, while eliminating the need to reprocess the entire dataset.

The unknown parameter vector  $\boldsymbol{\theta}$  is estimated through the minimization of a quadratic cost function, defined as follows:

$$V(\boldsymbol{\theta}) = \frac{1}{2} \sum_{k=1}^n [\mathbf{y}(k) - \boldsymbol{\varphi}^T(k)\boldsymbol{\theta}]^2 + \frac{1}{2} [\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(\mathbf{0})]^T \mathbf{P}^{-1}(\mathbf{0}) [\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(\mathbf{0})] \quad (2)$$

In this expression,  $\mathbf{P}(\mathbf{0}) \in \mathbb{R}^{p \times p}$  denotes the initial condition of the covariance matrix  $\mathbf{P}(k)$ , which is required to be positive definite. Concurrently,  $\hat{\boldsymbol{\theta}}(\mathbf{0})$  represents the initial estimate of the parameter vector.

The first term of the cost function quantifies the accumulated squared error between the observed outputs and the model predictions, whereas the second term penalizes deviations from the initial estimate, thereby enhancing numerical stability during the early stages of the algorithm.

Differentiating the cost function in equation (2) with respect to the parameter vector  $\boldsymbol{\theta}$  and equating the result to zero yields the following expression:

$$\theta = \left[ P^{-1}(0) + \sum_{k=1}^n \varphi(k) \varphi^T(k) \right] \left[ P^{-1}(0) \hat{\theta}(0) + \sum_{k=1}^n \varphi(k) y(k) \right] \quad (3)$$

This expression corresponds to the standard least squares algorithm. To derive its recursive formulation, a progressive update of the covariance matrix is introduced. Accordingly, the recursive estimation of the parameter vector is expressed as:

$$\hat{\theta} = P(k) \left[ P^{-1}(0) \hat{\theta}(0) + \sum_{i=1}^n \varphi(i) y(i) \right] \quad (4)$$

The recursive equations of the Recursive Least Squares algorithm for the scalar case can be summarized as follows:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-1) \varphi(k) e(k)}{1 + \varphi^T(k) P(k-1) \varphi(k)} \quad (5)$$

$$P(k) = P(k-1) - \frac{P(k-1) \varphi(k) \varphi^T(k) P(k-1)}{1 + \varphi^T(k) P(k-1) \varphi(k)} \quad (6)$$

$$e(k) = y(k) - \varphi^T(k) \hat{\theta}(k-1) \quad (7)$$

## 2.2. Construction of the EMG Model Using RLS.

To model the electromyographic (EMG) signal, experimental data are collected with a sampling period  $h$  and a total of  $N$  samples. These data are processed using the Recursive Least Squares algorithm described in equation (1), where  $y(k)$  denotes the acquired EMG signal,  $\theta$  represents the vector of unknown parameters, and  $\varphi(k)$  is the regression vector, defined as:

$$\varphi(k) = \sum_{j=1}^{100} [\sin(2\pi j t_k) f_j(t_k) + \cos(2\pi j t_k) f_j(t_k)] \quad (8)$$

where each function  $f_j(t_k)$  is defined as:

$$f_j(t_k) = \begin{cases} 1 & \text{if } j \in \{1, 2, 3, 4\} \\ t_k^{m_j} & \text{if } j > 4 \text{ and } j \text{ is even, with } m_j = \frac{j-4}{2} \\ \sqrt{t_k^{m_j}} & \text{if } j > 4 \text{ and } j \text{ is odd, with } m_j = \frac{j-4}{2} \\ e^{-t_k^{n_j}} \text{ or certain odd } j \text{ with } n_j = \frac{j-4}{4} \end{cases}$$

This formulation enables the construction of a rich functional basis, comprising sines and cosines modulated by polynomial, exponential, or root-type functions. The continuity and spectral diversity of this basis ensure sufficient persistence of the input signal, thereby facilitating robust estimation of the model parameters.

Once the RLS algorithm has been applied and the parameters  $\theta_i$  have been estimated, the corresponding EMG signal model can be constructed as:

$$EMG(t_k) = \sum_{i=1}^{200} \varphi_i(t_k) \theta_i \quad (9)$$

where  $\varphi_i(t_k)$  represents the value of the  $i$ -th regressor at instant  $t_k$ ,  $\theta_i$  is the corresponding estimated coefficient, and  $EMG(t_k)$  is the modeled signal.

In this way, the set of basis functions  $\varphi(t)$ , together with the parameter vector  $\varphi$ , constitute the complete mathematical model that allows reproducing the EMG signal from acquired data.

## 3. RESULTS

Following the acquisition and digitization of the electromyographic (EMG) signals, mathematical modeling was performed using parametric identification techniques. Specifically, the Recursive Least Squares (RLS) algorithm was employed due to its efficiency in online parameter estimation and low computational cost. The objective was to characterize the underlying dynamics of the EMG signals through discrete-time linear models capable of capturing their primary features.

For the validation phase, ten signals corresponding to various muscle movements were selected. These included simple actions, such as hand closure, as well as rapid contractions and movements requiring greater force. For demonstration purposes, three experimental results out of the ten are presented. For one of these, a complete analysis is provided, including four plots: the experimental EMG signal, the modeled

EMG signal, the comparison between both signals, and the resulting error. For the remaining two examples, only the comparison plots are shown, with a zoomed-in view of the regions where the signals differ, in order to optimize the clarity of the results.

All signals were acquired with a sampling period of 25 ms, each comprising 200 samples. This sampling interval was determined to be sufficient to capture the signal's variability, taking into account the processing limitations of the employed microcontroller. After acquisition, the signals were processed using the RLS algorithm to estimate the model parameters. Following the acquisition of the EMG signals, ten recordings corresponding to distinct types of muscle activity were selected for analysis. These encompassed both simple actions, such as hand opening and closing, as well as movements involving greater force or rapid execution with intense contractions. This dataset enabled the evaluation of the model's ability to accurately reproduce the EMG signals across a range of dynamic conditions.

Figure 1 illustrates the acquired EMG signal, consisting of a 5-second recording sampled at 200 points

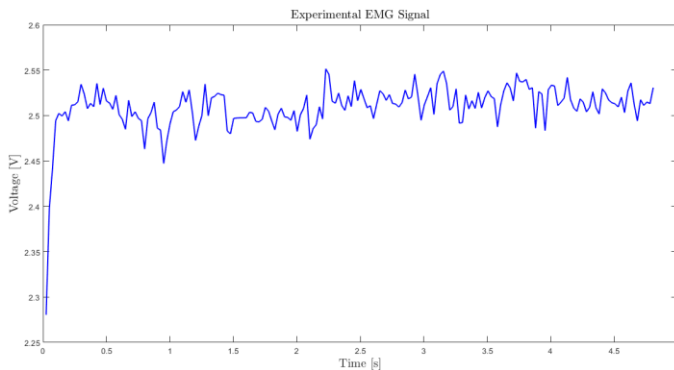


Figure 1. Experimental EMG Signal.

Figure 2 presents the modeled EMG signal, which, at first glance, closely resembles the original signal

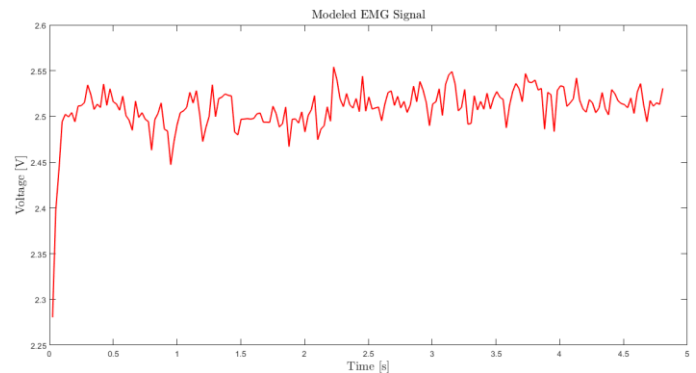


Figure 2. Modeled EMG Signal.

After obtaining the modeled EMG signal, a comparison with the original signal was performed to assess their similarity and identify any discrepancies. This comparison is shown in Figure 3.

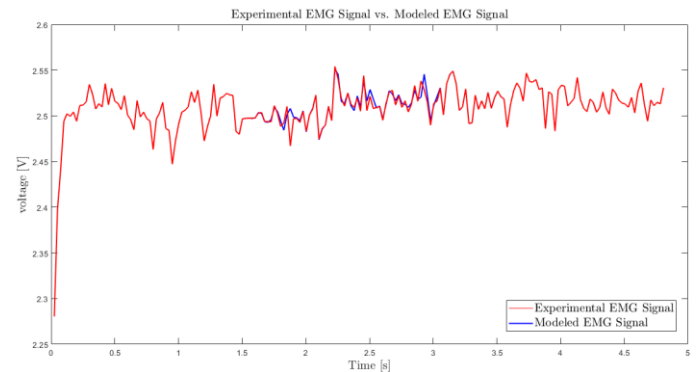


Figure 3. Experimental EMG signal vs Modeled EMG Signal.

In the signal comparison, a noticeable difference between the signals is observed between 1.7 and 3.3 seconds of the simulation; however, over the remainder of the recording, the signals appear nearly identical. To quantify this discrepancy, the error between the signals was calculated, as shown in Figure 4.

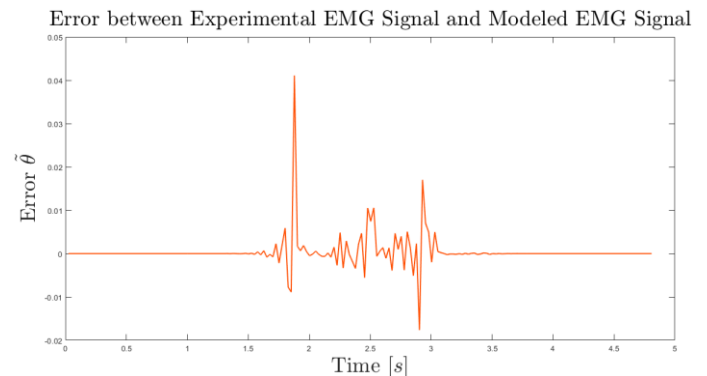


Figure 4. Error between experimental EMG signal and modeled EMG signal.

Analysis of the error between the signals indicates that it is small. Consequently, the model obtained through this identification can be considered highly accurate, as such a minimal error is negligible for the purposes of mechatronic system control.

For the following experimental signals, it is important to recall that these recordings were obtained from different forearm contractions, including rapid movements and high-force actions, which revealed distinct patterns in the EMG signals. Accordingly, Figures 5 and 6 present the experimental EMG signals compared with their modeled counterparts. The differences between the signals are highlighted in the lower part of each figure using a zoomed-in view, allowing clear visualization of the discrepancies, similar to the first signal analyzed. Nevertheless, these differences remain minimal, resulting in only a very small error between the signals.

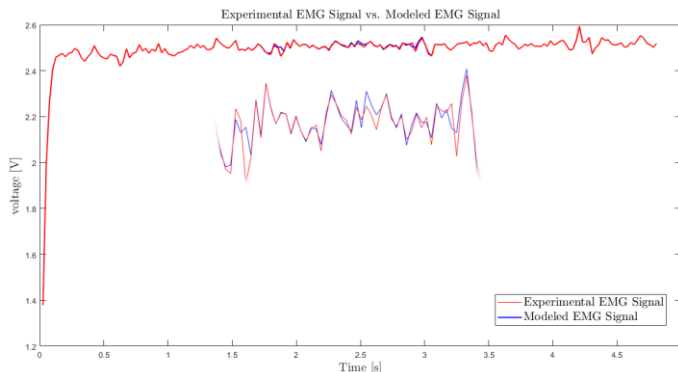


Figure 5. Difference between the experimental EMG signal and the modeled EMG signal.

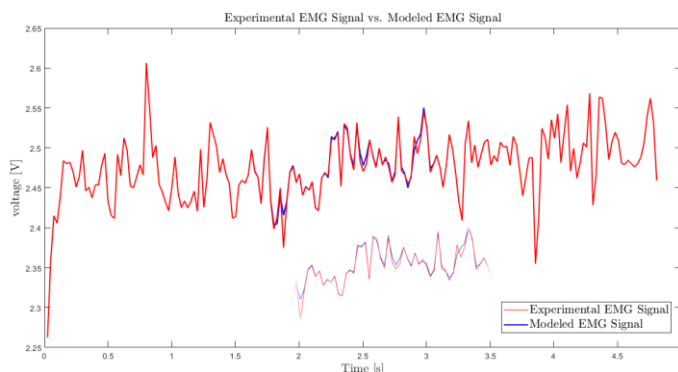


Figure 6. Difference between the experimental EMG signal and the modeled EMG signal.

In the context of the algorithm, it is important to emphasize the critical role of the Recursive Least Squares method, as it maintains dynamic updates of the parameters throughout the experiment until convergence to final values that enable accurate signal reproduction. This behavior is illustrated in Figure 7, which depicts the 200 terms being sequentially identified and varying over the course of the experiment. Such variation allows the determination of the optimal values to be applied with the proposed known functions, thereby achieving an accurate model of the EMG signal.

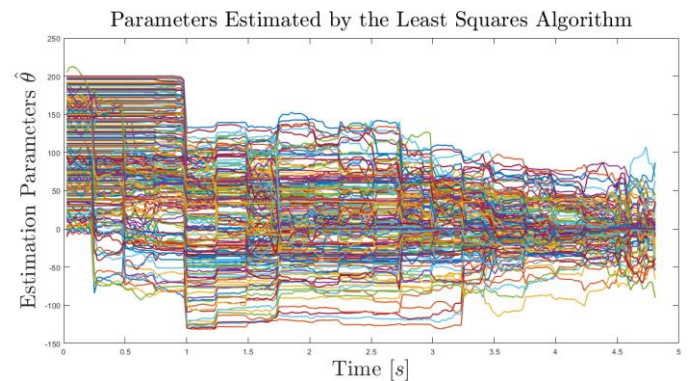


Figure 7. Estimated parameters of the EMG signal model.

This parameter estimation offers the advantage that, for any signal, the parameters are not constant but depend solely on the behavior of the signal being identified. This allows the estimation to closely approximate the true values of the experimental signal, leveraging mathematical functions to replicate the observed signal accurately. The benefit of the dynamic behavior inherent in the Recursive Least Squares algorithm is that, depending on the input signal's characteristics, the algorithm can efficiently identify the parameters without loss of information. Consequently, the parameter dynamics are illustrated in Figure 8. When converted to numerical form, Figure 8 presents a fragment of the estimated parameters obtained by the algorithm, enabling the model to accurately reproduce the desired signal.

|                     |                   |                     |                   |                     |                  |                     |                  |
|---------------------|-------------------|---------------------|-------------------|---------------------|------------------|---------------------|------------------|
| $\hat{\theta}_1$    | 11.831 140 654 5  | $\hat{\theta}_2$    | 1.355 823 686 1   | $\hat{\theta}_3$    | 2.058 006 217 8  | $\hat{\theta}_4$    | 6.390 348 316 9  |
| $\hat{\theta}_5$    | 4.697 106 117 3   | $\hat{\theta}_6$    | 6.862 985 310 5   | $\hat{\theta}_7$    | -1.404 950 685 1 | $\hat{\theta}_8$    | 1.669 886 394 7  |
| $\hat{\theta}_9$    | -25.665 554 569 2 | $\hat{\theta}_{10}$ | -32.183 976 056 7 | $\hat{\theta}_{11}$ | -2.447 857 605 9 | $\hat{\theta}_{12}$ | -4.619 189 158 1 |
| $\hat{\theta}_{13}$ | -6.772 604 060 1  | $\hat{\theta}_{14}$ | -3.527 877 740 9  | $\hat{\theta}_{15}$ | 2.870 227 403 0  | $\hat{\theta}_{16}$ | -1.463 072 386 5 |
| $\hat{\theta}_{17}$ | 31.210 264 121 8  | $\hat{\theta}_{18}$ | 0.421 148 480 4   | $\hat{\theta}_{19}$ | -2.436 786 468 0 | $\hat{\theta}_{20}$ | -0.678 955 123 7 |
| $\hat{\theta}_{21}$ | -25.469 284 509 3 | $\hat{\theta}_{22}$ | -7.133 584 518 9  | $\hat{\theta}_{23}$ | -0.105 139 222 3 | $\hat{\theta}_{24}$ | -0.426 741 057 5 |
| $\hat{\theta}_{25}$ | -5.470 047 788 4  | $\hat{\theta}_{26}$ | 0.971 770 463 4   | $\hat{\theta}_{27}$ | 2.104 077 579 2  | $\hat{\theta}_{28}$ | 4.690 145 619 4  |
| $\hat{\theta}_{29}$ | 0.114 103 260 3   | $\hat{\theta}_{30}$ | 1.361 682 953 4   | $\hat{\theta}_{31}$ | 0.033 898 064 8  | $\hat{\theta}_{32}$ | -1.782 116 141 8 |
| $\hat{\theta}_{33}$ | 7.848 739 546 1   | $\hat{\theta}_{34}$ | -12.633 768 637 2 | $\hat{\theta}_{35}$ | 2.224 093 293 9  | $\hat{\theta}_{36}$ | -4.741 329 145 6 |
| $\hat{\theta}_{37}$ | 9.716 331 715 2   | $\hat{\theta}_{38}$ | -3.951 324 294 9  | $\hat{\theta}_{39}$ | -1.248 339 622 9 | $\hat{\theta}_{40}$ | 0.623 494 397 8  |
| $\hat{\theta}_{41}$ | -19.882 216 265 8 | $\hat{\theta}_{42}$ | 11.131 205 278 3  | $\hat{\theta}_{43}$ | 3.414 823 030 5  | $\hat{\theta}_{44}$ | 1.648 084 679 1  |
| $\hat{\theta}_{45}$ | -1.189 208 677 8  | $\hat{\theta}_{46}$ | -0.963 593 283 5  | $\hat{\theta}_{47}$ | 0.046 064 374 8  | $\hat{\theta}_{48}$ | 0.033 007 892 6  |

Figure 8. Numerical representation of the estimated parameters of the Recursive Least Squares algorithm..

In this initial stage of developing the mathematical model, for practical metric purposes, the difference between the experimental EMG signal and the modeled EMG signal was used. Although the resulting error is very small, it is significant because, beyond merely obtaining a model, its mathematical structure is well-founded using studied functions. At the same time, the recursive algorithm is easily implementable in embedded systems, as it can be programmed in any programming language without requiring specialized libraries or software. This presents advantages in applications such as minimizing the time a patient must spend during prosthesis development and avoiding invasive or lengthy procedures each time EMG signals need to be acquired. Moreover, it facilitates the training of AI algorithms for predicting neurological disorders and enables real-time digital control applications, as this model provides the necessary information to process EMG signals under any conditions.

In comparison to models available in the literature, many focus on statistical analysis, such as Gaussian distributions, to generate a generic EMG signal that satisfies the conditions of a real EMG signal but requires high computational cost. Other models aim to replicate the generation of EMG signals based on the potentials acquired along the central and peripheral nervous system to the muscle region, creating the superposition of signals. However, very few approaches have successfully developed a recursive model capable of producing an EMG signal directly from the raw EMG data.

#### 4. CONCLUSIONS

This study demonstrates that the Recursive Least Squares (RLS) algorithm is a highly effective and computationally efficient method for modeling electromyographic (EMG) signals, particularly those arising from forearm muscle activity. By acquiring and processing real EMG recordings, the RLS algorithm adaptively estimates the model parameters in real time, with the parameters dynamically reflecting the evolving behavior of the input signal rather than remaining constant. This dynamic adaptation allows the model to closely approximate the true experimental signal, leveraging well-defined mathematical functions to replicate its characteristics accurately.

Comparisons between the experimental and modeled signals revealed minimal differences, with error analysis confirming that discrepancies are negligible for practical applications in mechatronic system control. The recursive nature of RLS enables continuous parameter updates throughout the

experiment, converging to optimal values and maintaining robust signal reproduction. Moreover, the algorithm's simplicity and low computational cost make it easily implementable in embedded systems without the need for specialized software or libraries.

These advantages position RLS as a powerful tool for real-time biomedical signal modeling, with significant potential in applications such as rehabilitation engineering, prosthetic control, human-machine interfaces, and the training of AI algorithms for neurological assessment. Its capability to generate accurate, dynamically adapted models from raw EMG data highlights its superiority over traditional statistical or physiologically derived EMG models, which often require high computational effort or do not support recursive, real-time implementation.

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