

DESIGN OF A LQR BY A WOA APPLIED TO THE POSITION REGULATION OF A PIVOTED HELICOPTER

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ABSTRACT

One application area of Artificial Intelligence is the design of position regulators for pivoted helicopters. These are commonly controlled through optimal control techniques such as Linear Quadratic Regulators (LQR), which are susceptible to optimization using bio-inspired algorithms. These algorithms enable efficient solution searches in the parameter space to effectively fine-tune the regulators. This research focuses on the intelligent optimization of an LQR for the control of a pivoted helicopter, using a Whale Optimization Algorithm. The proposed algorithm significantly minimizes the control performance indices compared to the analytical tuning method and a Genetic Algorithm.

Keywords: Optimization, pivoted helicopter, Linear Quadratic Regulator, Whale Optimization Algorithm.

RESUMEN.

Un área de aplicación de la Inteligencia Artificial es el diseño de reguladores de posición de helicópteros pivotados. Estos comúnmente son controlados por medio de técnicas de control óptimo como los Reguladores Lineales Cuadráticos (LQR), los cuales son susceptibles de optimizarse por medio de algoritmos bio-inspirados. Estos algoritmos permiten la búsqueda eficiente de soluciones en el espacio de parámetros para afinar los reguladores de manera efectiva. Este trabajo de investigación se centra en la optimización inteligente de un LQR para el control de un helicóptero pivotado, por medio de un Algoritmo de Optimización de Ballenas (WOA). Este algoritmo propuesto disminuye significativamente los índices de desempeño del control, con respecto del método de ajuste analítico y un Algoritmo Genético.

Palabras Clave: Optimización, helicóptero pivotado, Regulador Cuadrático Lineal, Algoritmo de Optimización de Ballenas.

1. INTRODUCTION

A pivoted helicopter is a scaled-down platform that, unlike a real helicopter, is restricted to two degrees of freedom (DoF). This is achieved by anchoring the model to a vertical axis (see Figure 1), which limits the aircraft's movement to elevation, around the pitch axis (θ), and yaw control, around the yaw axis (ψ). This configuration allows for the exploration of relevant concepts and principles regarding aircraft control and flight dynamics [1].

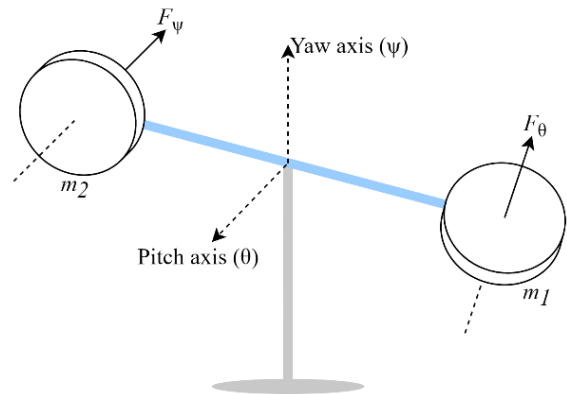


Figure 1. Free body diagram of the 2-DoF helicopter.

The flight dynamics of the helicopter can be described as follows: the helicopter is in a horizontal position when $\theta = 0$. The pitch angle increases positively ($\dot{\theta}(t) > 0$) when the nose (m_1) rises. The yaw angle increases positively ($\dot{\psi}(t) > 0$) when the body rotates clockwise. Additionally, $\theta > 0$ if the pitch thrust force is positive ($F_\theta > 0$), while the yaw angle is positive ($\psi > 0$) when the yaw thrust force is positive ($F_\psi > 0$) [1]. One challenge presented by this topology of Unmanned Aerial Vehicle (UAV) is the regulation of its position. This becomes the focus of study in the research process documented in this proposal.

2. THEORETICAL FRAMEWORK

In the specialized literature, numerous studies have been documented focusing on the design of controllers using the pivoted helicopter as a testbed to analyze the effectiveness of various control strategies. These include robust control, intelligent control, and optimal control.

Within robust control, techniques such as sliding mode control [2], Super-Twisting Control [3], and adaptive control [4] have been extensively explored. In the realm of intelligent control, approaches using neural networks [5] and fuzzy logic [6] have been employed. Optimal control has seen the use of Integral Quadratic Regulators (LQI) [7] and Linear Quadratic Regulators (LQR) [8].

It is important to highlight that in the state-of-the-art literature, these control strategies have benefited from the use of Artificial

Intelligence techniques to tune parameters and enhance the effectiveness of pivoted helicopter control. For instance, Genetic Algorithms, Ant Colony Optimization, and Particle Swarm Optimization have been used for optimal tuning of controllers [9][10].

2.1. Linear Quadratic Regulator

The control techniques reported in the specialized literature have been classified into classical control and modern control. Classical techniques are based on the analysis of single-input single-output (SISO) systems in the frequency domain, suitable for handling linear and low-order systems. In contrast, modern techniques deal with multi-input multi-output (MIMO) systems in the time domain, used to manage complex dynamics. An example of the latter is the Linear Quadratic Regulator (LQR). An LQR minimizes a quadratic cost function of a linear dynamic system, defined by equation 1.

$$J = \int_0^{\infty} (\vec{x}^T Q \vec{x} + \vec{u}^T R \vec{u}) dt = Cx \quad (1)$$

Where $\vec{x} \in \mathbb{R}^n$ is the state vector of the system at time t , $\vec{u} \in \mathbb{R}^m$ is the control input vector, $Q \in \mathbb{R}^{n \times n}$ is a positive semi-definite symmetric matrix that penalizes deviations in the system states, and $R \in \mathbb{R}^{m \times m}$ is a positive definite symmetric matrix that weights the magnitude of the control inputs [11].

The objective of LQR is to find the control law of equation 2, which minimizes the cost function J of equation 1 [12].

$$\vec{u} = -K\vec{x} \quad (2)$$

Where the optimal gain matrix, K is computed using equation 3 [12].

$$K = R^{-1}B^T P \quad (3)$$

And P is a symmetric positive definite matrix that satisfies the algebraic Riccati equation [12], defined as: $A^T P + PA - PBR^{-1}B^T P + Q = 0$.

The general scheme of a system controlled by an LQR is shown in Figure 2. A , B and C are matrices that model the dynamics of the system and K is the gain matrix of equation 3 [13].

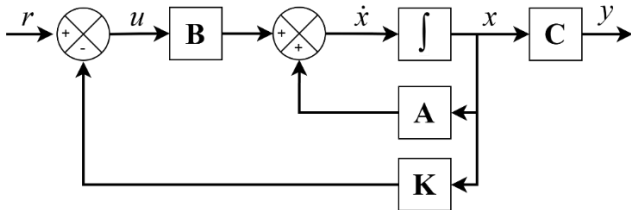


Figure 2. LQR Control Scheme.

In the design of an LQR controller, the choice of the Q and R matrices is essential to achieve the desired performance of the

controlled system. These matrices are parameters that directly influence the behavior of the controller, affecting aspects such as stability, robustness and response of the system. The Q matrix determines the relative importance of each state variable in the cost function that the LQR minimizes. While the R matrix determines the intensity of control actions [12]. The form of these matrices is defined as:

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{bmatrix}, R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix}.$$

Proper determination of the values of the Q and R matrices is approached classically, choosing values based on experience and knowledge of the system, and then adjusting them through trial and error. Or, analytically, calculating the matrix Q by multiplying C by its transpose C^T and defining the matrix R as an identity matrix. An alternative to this is to use optimization techniques to explore the solutions space and find the most suitable values for Q and R .

2.2. Optimization

To define the concept of optimization, it is necessary to consider a mathematical function $f(\vec{X}) \forall \vec{X} = \{x_1, x_2, x_3, \dots, x_n\} \in \mathbb{R}/n \in \mathbb{Z}$ is the number of decision variables of the problem represented by f . Therefore, optimization is defined mathematically as $\min_{\vec{X} \in \mathbb{R}} f(\vec{X}) : g_k(\vec{X}) \leq 0, k = 1, 2, \dots, m \wedge$

$$h_k(\vec{X}) = 0, k = 1, 2, \dots, r \forall m, r < n.$$

Where $g_k(\vec{X})$ and $h_k(\vec{X})$ are functions called constraints [14]. When $\vec{X} \in \mathbb{R}$ it is a numerical optimization problem, while for $\vec{X} \in \mathbb{Z}$ is a combinatorial optimization problem. So that, $\vec{X}^*$ represents the vector that maximizes or minimizes f , this is known as the optimal solution [15].

To find $\vec{X}^*$ in an optimization problem such that $\vec{X}^* \in \mathbb{R}$, various alternatives are discussed in the specialized literature, grouped into stochastic methods and deterministic methods [15]. The latter are techniques that allow finding the best solution to problems where the same inputs produce the same outputs [16]. Examples include linear programming, nonlinear programming, and non-convex nonlinear programming, among others.

An alternative to deterministic techniques for solving numerical optimization problems are the stochastic methods. These are used to solve optimization problems in which there are factors that depend on randomness [16]. A series of stochastic methods are bio-inspired algorithms, which imitate processes and patterns that can be found in nature such as biological processes, behaviors and structures of living organisms.

2.3. Whale Optimization Algorithm

The Whale Optimization Algorithm (WOA) is an optimization technique inspired by the hunting behavior of humpback whales,

proposed by Seyedali Mirjalili and Andrew Lewis in 2016. The WOA is based on the principles of the feeding strategy of humpback whales, known as bubble-net feeding. This involves the leading whale finding the prey, diving a few meters, creating spiral bubbles around its food, and then swimming to the surface to attack [17], as illustrated in Figure 3.

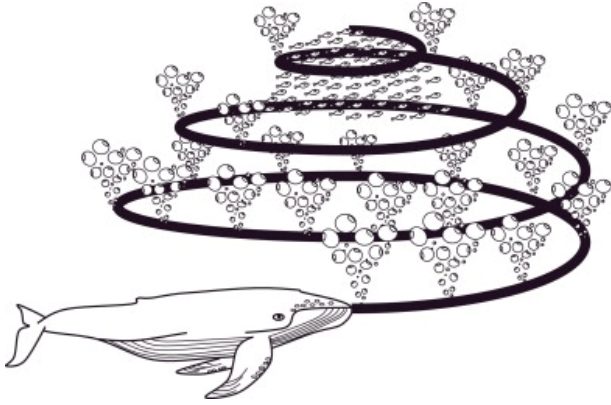


Figure 3. Bubble-net feeding.

2.3.1 Search for prey

This algorithm begins with the generation of an initial group of possible solutions, called search agents (whale positions), represented as:

$$\vec{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{in} \end{bmatrix},$$

where, i is the number of agents and n is the dimension of the problem. The initial population of i search agents is generated randomly within the bounds of the problem's universe of discourse, given by $[x_{min}, x_{max}]$ for each x_{in} in $\vec{X}$. The best current solution $\vec{X}^*$, within $\vec{X}$, is considered the prey. This is obtained by calculating the fitness of each search agent $\vec{X}_i$ in the problem represented by the function $f(\vec{X}_i)$ [18].

2.3.2 Encircling prey

The WOA consists of two consecutive phases that model the hunting behavior of humpback whales. It begins with the encircling method, where the whales identify and surround their prey. Once the prey $\vec{X}^*$ is identified, the rest of the search agents move towards it to encircle it. The position update of each search agent $\vec{X}_i$ is carried out using equations 4 and 5 [17].

$$\vec{D} = |\vec{C} \cdot \vec{X}^*(t) - \vec{X}_i(t)| \quad (4)$$

$$\vec{X}_i(t+1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D} \quad (5)$$

Where $\vec{A}$ and $\vec{C}$ are coefficient vectors calculated as $\vec{A} = 2\vec{a} \cdot \vec{r} - \vec{a}$ and $\vec{C} = 2\vec{r}$ whit $\vec{a} = 2 - \frac{2t}{t_{max}}$, decreasing from 2 to 0 during the iterative process, $\vec{r}$ being a vector of random numbers in the range $[0, 1]$, and t is the current iteration number.

2.3.3 Bubble-net attacking method

The next phase focuses on using a bubble-net strategy to attack the prey, combining two approaches. The first approach, known as the *Shrinking encircling mechanism*, is implemented by adjusting the value of $\vec{A}$ in equation 5, where a random value α is assigned from $[-1, 1]$. This allows the new position of a search agent $\vec{X}_i(t+1)$ to be defined anywhere between the original position of the agent $\vec{X}_i(t)$ and the position of the current best agent $\vec{X}^*(t)$ [17].

The second approach, known as *Spiral updating position*, simulates the behavior of humpback whales as they spiral around their prey [17]. Equation 6 represents this movement. Where $\vec{D} = |\vec{X}^*(t) - \vec{X}_i(t)|$ and l is a random number in the range $[-1, 1]$.

$$\vec{X}_i(t+1) = \vec{D} \cdot e^l \cdot \cos(2\pi l) + \vec{X}^*(t) \quad (6)$$

Here, the whales move around the prey in a progressively smaller circle while following a spiral trajectory. To simulate this behavior, the probability p_{rand} of choosing one of the two methods described earlier is set at 50% [17] [18]. Thus, the mathematical model for the swimming behavior of each whale can be represented as in equation 7.

$$\vec{X}_i(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A} \cdot \vec{D}; & p < 0.5 \\ \vec{D} \cdot e^l \cdot \cos(2\pi l) + \vec{X}^*(t); & p \geq 0.5 \end{cases} \quad (7)$$

The WOA cyclically repeats these phases for a maximum number t_{max} of times and terminates when the termination criterion is met, when $t = t_{max}$. This process is depicted in the flowchart shown in Figure 4 [18].

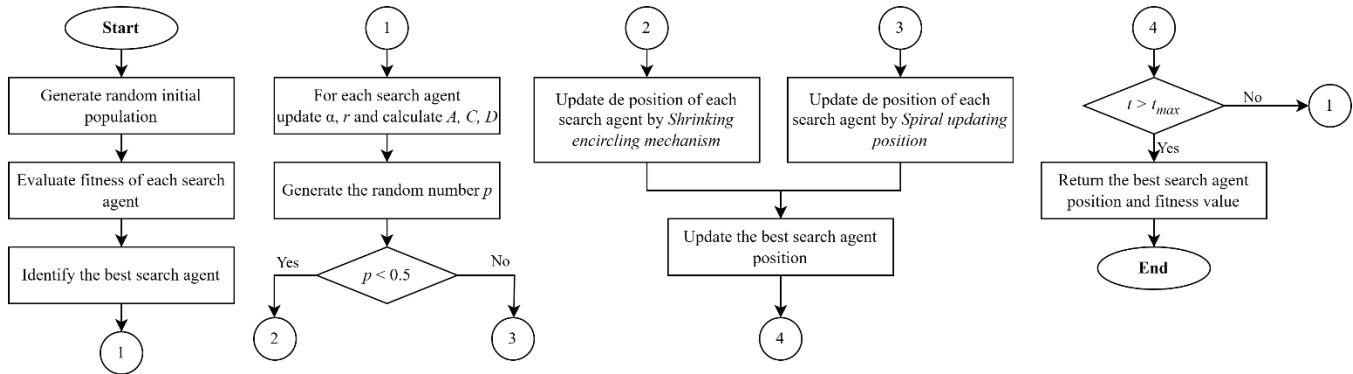


Figure 4. Whale Optimization Algorithm flowchart.

3. METHODOLOGY

In this section, the proposed methodology for the research is described. As depicted in the schematic in Figure 5, firstly, the mathematical model of the helicopter is obtained. Using this model, an LQR controller is designed through an analytical approach. Subsequently, the Whale Optimization Algorithm (WOA) is applied to tune the controller parameters, alongside using a Genetic Algorithm (GA) for the same purpose. Finally, the controller is validated, and a comparative study of the results obtained is conducted.

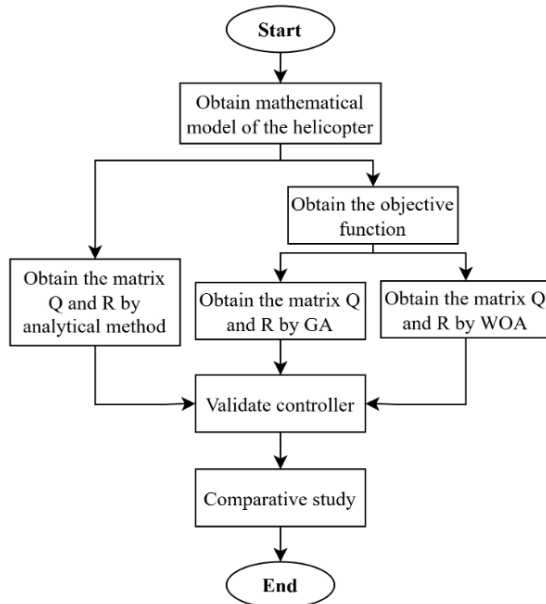


Figure 5. Methodological Scheme.

The dynamic behavior of the pivoted helicopter, with p inputs, q outputs y n state variables, is mathematically described by the state equation 8 and the output equation 9[1].

$$\dot{\vec{x}}(t) = A\vec{x}(t) + B\vec{u}(t) \quad (8)$$

$$\vec{y}(t) = C\vec{x}(t) + D\vec{u}(t) \quad (9)$$

Where, $\vec{x} \in \mathbb{R}^n$ represents the state vector, $\vec{u} \in \mathbb{R}^p$ is the input vector, $\vec{y} \in \mathbb{R}^q$ is the output vector and $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times p}$, $C \in \mathbb{R}^{q \times n}$, $D \in \mathbb{R}^{q \times p}$ are the state matrices of the helicopter. Table 1 shows the values of these matrices [1].

Matrix	Values
A	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9.2768 & 0 \\ 0 & 0 & 0 & -0.34931 \end{bmatrix}$
B	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2.3667 & 0.0789 \\ 0.2406 & 0.7909 \end{bmatrix}$
C	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
D	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Given the state-space representation of the pivoted helicopter, the matrices Q and R , are defined as follows:

$$Q = \begin{bmatrix} q_{11} & 0 & 0 & 0 \\ 0 & q_{22} & 0 & 0 \\ 0 & 0 & q_{33} & 0 \\ 0 & 0 & 0 & q_{44} \end{bmatrix}, \quad R = [r_{11}],$$

Thus, q_{11} , q_{22} , q_{33} , q_{44} and r_{11} , constitute each search agent $\vec{X}_i$ in the WOA and each chromosome in the GA. The optimization function is denoted by equation 10. This function aims to

minimize the sum of the integral and quadratic errors of the helicopter's response over time.

$$F_{obj} = \frac{1}{(IAE + ISE + ITAE + ITSE) + 1} \quad (10)$$

Where IAE stands for Integral Absolute Error, ISE for Integral Square Error, ITAE for Integral Time Absolute Error, and ITSE represents Integral Time Square Error. The addition of 1 in the denominator prevents division by zero. The parameters of the algorithms used are shown in Table 2.

Table 2. Optimization Algorithm Parameters.

Parameter	Values
Search space	[0, 10]
Variables	$q_{11}, q_{22}, q_{33}, q_{44}, r_{11}$
Number of iterations	60
Population size	100

4. RESULTS

This section discusses the results obtained from this proposal for tuning the parameters of an LQR applied to the position control of a pivoted helicopter, through a Whale Optimization Algorithm.

Firstly, Figure 6 shows an example of the WOA vs GA convergence curves. This pattern suggests that the WOA was able to explore the search space faster and better, finding solutions that minimized the sum of the errors in the helicopter's response to a step reference signal, with respect to the GA. These results are summarized in Table 3.

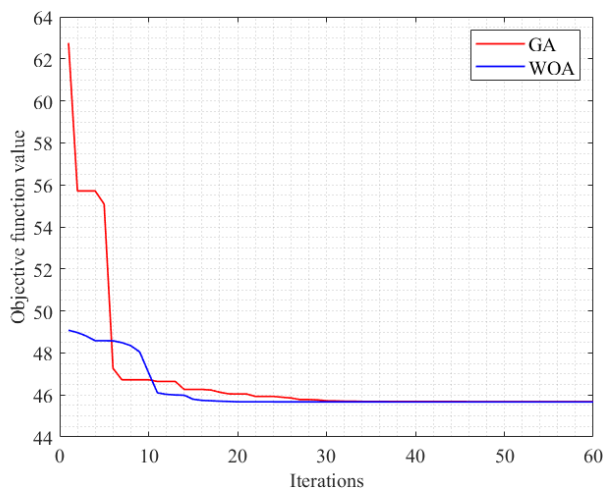


Figure 6. Comparison of convergence curves of WOA and GA.

Table 3. Comparison of convergence of WOA and GA.

Parameter	WOA	GA
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Objective function value	45.6680	45.7852
Error sum reduction %	70.0997%	70.0229%
Iteration	18	35

The algorithms were executed 40 times to carry out a repeatability study of the sum of the errors. Table 4 shows the values of the Kolmogorov normality test, which indicates that the algorithms do not follow a normal distribution ($p < 0.05$). Therefore, the Friedman test was used, which indicates significant differences between the algorithms. In addition, the median and interquartile range were calculated to analyze the distribution and dispersion of the data, determining that the WOA, having a lower median and interquartile range of the sum of the errors, stands out as the best option.

Table 4. Statistical results of the sum of errors.

	WOA	GA
Kolmogorov test	1.08×10^{-36}	1.03×10^{-36}
Friedman test	9.03×10^{-6}	
Median	45.6785	45.7710
Interquartile range	0.0365	0.3945

The solution matrices are the product of the median of the best search agents in the 40 repeatability test runs. This allows tuning the Q and R matrices, as shown in expression 11, achieving a significant improvement in the performance of the LQR, to considerably improve the stability and precision of the control of the pivoted helicopter, with respect to the analytical adjustment. The results are presented in Figure 7 and summarized in Tables 5 and 6.

$$Q = \begin{bmatrix} 5.3837 & 0 & 0 & 0 \\ 0 & 9.9999 & 0 & 0 \\ 0 & 0 & 0.1000 & 0 \\ 0 & 0 & 0 & 0.1000 \end{bmatrix} \quad R = [7.7306] \quad (11)$$

Table 5. Comparison of Pitch axis performance criteria.

Performance criterion	LQR	LQR+WOA
Settling time	18 s	22 s
Steady state error	-12 %	-0.09 %
IAE	7.3968	3.6713
ISE	2.3208	1.9519
ITAE	141.9408	12.4667
ITSE	18.0214	3.3187

Table 6. Comparison of Yaw axis performance criteria.

Performance criterion	LQR	LQR+WOA
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Settling time	20 s	18 s
Steady state error	13 %	-0.56 %
IAE	9.4479	4.0475
ISE	2.9256	2.0190
ITAE	173.5226	19.4961
ITSE	26.1196	3.9147

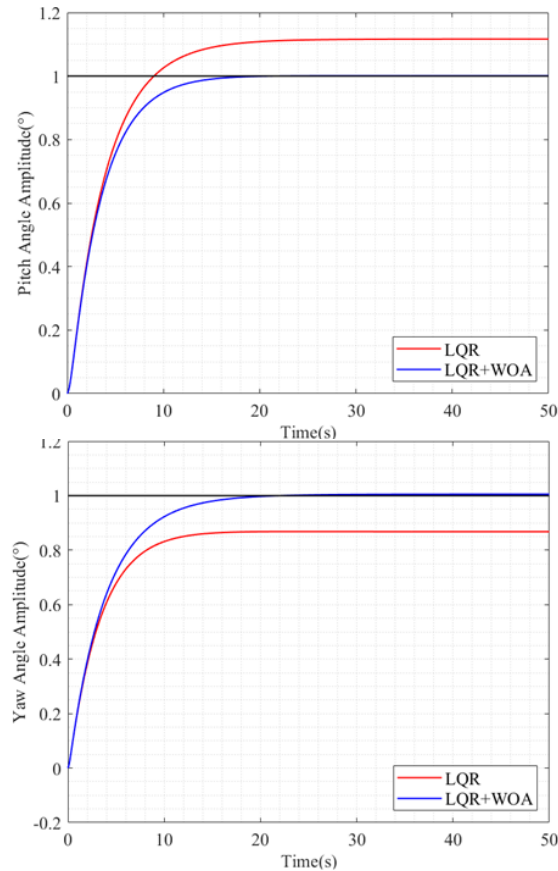


Figure 7. Response of the LQR vs LQR tuned by the WOA.

5. CONCLUSIONS

According to the results of the simulations carried out, it is observed that it is possible to tune the values of the Q and R matrices of a Quadratic Linear Regulator applied to the position control of a pivoted helicopter through a Whale Optimization Algorithm. Consequently, the LQR not only stabilizes the plant but also minimizes the performance criteria. In other words, setup times are significantly improved and errors for each axis are minimized, compared to traditional analytical tuning. As future work, it is proposed to compare the results obtained with other bio-inspired algorithms, which could provide a deeper understanding of the application of different optimization techniques in the tuning of the LQR controller applied to the pivoted helicopter. Additionally, it is suggested to investigate the

use of a hybrid control method, such as a PID+LQR controller, which could further improve response times and control accuracy, providing a more robust and efficient solution for helicopter position control.

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